Summary for computer simulations of charged systems
Scaling Description of Charged Polymers

In this paper, some assumptions and simplification:
(1) Use linear Debye-Huckel approximation \( \nu(r) = kT \frac{I_a}{r} \exp(-\kappa r) \).
(2) All the small ions are point-like
(3) Ignore the specificity of the counterions
(4) Debye-Huckel theory of simple electrolytes can be used, i.e., the concentration of small ions is not too large

1. Weakly charged polyelectrolytes in dilute solutions
Problem: Flexible weakly charged polyelectrolytes with a fraction of charged monomers \( f \ll 1 \).

(1) Flory theory and electrostatic blobs
Some shortcomings:
 a. Ignore the fact that the electrostatic interactions are long ranged and that monomers inside each blob interact with all the monomers of the chain.
 b. It implicitly assumes that the tension is constant along the chain, this is not true since the electrostatic potential is higher at the center of the chain than close to the end points.

(2) Polyelectrolytes in a poor solvent: the pearl-necklace model

(3) Annealed polyelectrolytes
The chemical potential of the charges \( \mu \) is fixed by the pH of the solution.

(4) Polyelectrolyte stretching and polyelectrolyte gels
In a \( \theta \) solvent, a polyelectrolyte acts as a Gaussian spring with a spring constant.
In a poor solvent, the elasticity of the pearl-necklace structure can be studied by minimizing its free energy in the presence of an external force.

2. Strongly charged polyelectrolytes and counterion condensation

(1) Non linear electrostatics and ion condensation
Can be modeled as rods.
In this case, Poisson-Boltzmann equation can be solved using a cell model where each rodlike polymer is embedded in a cylindrical cell that is electrically neutral.

(2) Beyond the Poisson-Boltzmann approximation, attractive interactions
At "high temperatures" one can consider the thermal fluctuations in the density of condensed counterions along the rod and treat them perturbatively.

3. Semidilute solutions